Chapter 3
Powers and Exponents

Objectives:
- to use powers to represent repeated multiplication
- to solve problems involving powers
3.1 – Using Exponents to Describe Numbers

This reads as, “3 to the power of 4” or “3 to the fourth power.” Exponents are used as a short-cut method to show how many times a number is multiplied by itself.

\[ 3^4 = 3 \times 3 \times 3 \times 3 \]  (note: that the • symbol can be substituted for the × symbol)

\[ 3^4 = 81 \]

The base can also be a negative number:

\[ (-3)^4 \]

\[ (-3)^4 = -3 \cdot -3 \cdot -3 \cdot -3 \]  (even number of negative signs means answer is positive)

\[ (-3)^4 = 81 \]

NOTE:
Whenever you have a negative base and the exponent is even, your answer will always be ______________.

Whenever you have a negative base and the exponent is odd your answer will always be ______________.

Example:  (a) \((-2)^4\) \hspace{2cm} (b) \((-2)^3\)
Note: The parenthesis must stay around the (-2) to indicate that (-2) is raised to the fourth power. Without the parenthesis you would get a different answer.

Example: \(-2^4\)
Since -2 is not in parenthesis, this problem really means: \(-1 \cdot 2^4\)

\[-2^4 = (-1) \cdot 2 \cdot 2 \cdot 2 \cdot 2\]

\[-2^4 = -16\]

\[2 \cdot 2 \cdot 2 \cdot 2 = 16\]
\[16(-1) = -16\]

Tip! When you have a 0 as an exponent, your answer will always be ______. The only exception is \(0^0 = ____\).

Let’s look at why that is:

<table>
<thead>
<tr>
<th>Power</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3^4)</td>
<td></td>
</tr>
<tr>
<td>(3^3)</td>
<td></td>
</tr>
<tr>
<td>(3^2)</td>
<td></td>
</tr>
<tr>
<td>(3^1)</td>
<td></td>
</tr>
<tr>
<td>(3^0)</td>
<td></td>
</tr>
</tbody>
</table>

Practice using a calculator to evaluate powers:

1. \(2^8 =\)
2. \(10^{12} =\)
3. \(6^9 =\)
4. \(7.5^5 =\)
5. \((-3)^8 =\)
6. \((-9)^3 =\)
7. \((-3.2)^3 =\)
8. \(-5^4 =\)
9. \(-4^3 =\)

Homework: “Power to the Exponent” and “Exponents and Powers” Worksheet

Workbook: Pages 30 – 31 #1 – 5, 7 – 13
3.2 – Exponent Laws

We use exponent laws to simplify expressions and make evaluating powers easier to calculate.

Product of Powers: \[ a^m \times a^n = a^{m+n} \]

Quotient of Powers: \[ \frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0) \]

Power of a Power: \[ (a^m)^n = a^{mn} \]

Power of a Product: \[ (ab)^n = a^n b^n \]

Power of a Quotient: \[ \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \]

Zero exponent: \[ a^0 = 1 \quad (a \neq 0) \]

Negative exponent: \[ a^{-n} = \frac{1}{a^n} \quad (a \neq 0) \]

Examples:

- **Multiplying Powers with the Same Base**

  Rule: add the exponents

  Method 1: Use Repeated Multiplication

  \[ 2^3 \times 2^2 = \]

  Method 2: Apply the Exponent Laws

  \[ 2^3 \times 2^2 = \]
Examples:
8^5 \times 8^4 = a^6 \times a = 7^2 \times 7^a = 7^6

• Divide Powers with the Same Base

Rule: subtract the exponents

Method 1: Use Repeated Multiplication

(-5)^9 \div (-5)^4 =

Method 2: Apply the Exponent Laws

(-5)^9 \div (-5)^4 =

Examples:
9^{16} \div 9^7 = \frac{9^9}{3^3} = \frac{a^{10}}{a^4} = \frac{x^6}{x^3} = x^3

Practice of multiplying and dividing exponents: Simplify and evaluate where possible.

a) y^7 \times y^{12} = \quad b) 5^3 + 5^2 = \quad c) 5^{17} \div 5^{10} = \quad d) a^{20} \div a^4 =

e) (-6)^{10} \div (-6)^5 =

f) \frac{(-7)^{10}}{(-7)^4} = \quad g) \frac{2^7 \times 2^4}{2^8} =
• **Raise Powers**
  
  Rule: multiply the powers
  
  Method 1: Use Repeated Multiplication
  
  \[(2^3)^2 = \]
  
  Method 2: Apply the Exponent Laws
  
  \[(2^3)^2 = \]
  
  Examples:
  
  \[(a^{10})^4 = \quad (3^2)^2 = \quad (-1)^6)^3 = \]
  
  Practice: \[(4^2)^5 = \quad (x^3)^3 = \quad (m^4)^3 = \quad (-2^3)^5 = \]
  
  **Homework:** “The Exponent Rules” and “Working with Exponents” Worksheet
  
  **Continuing Exponent Laws:**
  
  • **Products and Quotients to an Exponent**
    
    Rule: Raise power to each of the numbers
    
    Method 1: Use Repeated Multiplication
    
    (a) \([2 \times (-3)]^4\) \quad (b) \((\frac{3}{4})^3\)
Method 2: Apply the Exponent Laws

(a) \([2 \times (-3)]^4\)  

(b) \((\frac{3}{4})^3\)

Examples: \((3^2 \times 3^0)^2 = (3ab)^3 = (a^2 b^5)^3 = (-2mn)(-4m^3 n^2) =\)

Note: \((5 + 4)^3 \neq 5^3 + 4^3\)

• Exponent of Zero

Examples: \(\frac{3^4}{3^4} = (b) 2^{-3} \times 2^3\)

Practice: 
(a) \([7 \times (-2)]^3\)  
(b) \((\frac{2}{5})^4\)  
(c) \((2xy)^3\)  
(d) \(2ab(-3ab)\)  
(e) \(\frac{a^3 \times a^2}{a^5}\)
- **Negative Exponent**

Using your calculator determine the power of $2^{-2}$ _________________.

Now change your answer to a fraction.

How would you be able to determine the power of $2^{-2}$ without a calculator?

The negative exponent becomes **positive** once the base is reciprocated.

Examples: $(-5)^3$  \[ \left(\frac{2}{3}\right)^{-2} \]  \[ \frac{5^{-2}}{3^{-4}} \]  \[ \frac{1}{2^3} = 2^{-3} \]  \[ \text{Examples: (5} \]

**Homework:** *Workbook:* Pages 32 – 33 #1 – 12 and “*Exponent Law*” Worksheet
3.3 – Order of Operations

Problem: Evaluate the following arithmetic expression:
3 + 4 x 2

Solution:

<table>
<thead>
<tr>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 + 4 x 2</td>
<td>3 + 4 x 2</td>
</tr>
<tr>
<td>= 7 x 2</td>
<td>= 3 + 8</td>
</tr>
<tr>
<td>= 14</td>
<td>= 11</td>
</tr>
</tbody>
</table>

It seems that each student interpreted the problem differently, resulting in two different answers. Student 1 performed the operation of addition first, then multiplication; whereas student 2 performed multiplication first, then addition. When performing arithmetic operations there can be only one correct answer. We need a set of rules in order to avoid this kind of confusion. Mathematicians have devised a standard order of operations for calculations involving more than one arithmetic operation.

Rule 1: First perform any calculations inside parentheses.

Rule 2: Next evaluate any exponents

Rule 3: Then perform all multiplications and divisions, working from left to right.

Rule 4: Lastly, perform all additions and subtractions, working from left to right.

Therefore, Student ____ was correct because the rules were followed in the correct order.

In order to solve a question with multiple operations (add/subtract, multiply/divide) there is an order to follow often referred to as “BEDMAS”.

**BEDMAS** is an acronym that stands for:

- **B** – Brackets
- **E** – Exponents
- **DM** – Multiply or divide (left to right)
- **AS** – Add subtract (left to right)

This acronym is designed to help you remember what order to do the work in.
Examples:

(a) $3(2)^4$  
(b) $-3(-5)^2$  
(c) $4^2 + (-4^2)$

(d) $4^2 - 8 \div 2 + (-3^2)$  
(e) $(3 + 6) - 8 \times 3 \div 24 + 5$

(f) $8(5 + 2)^2 - 12 \div 2^2$  
(g) $-2(-15 - 4^2) + 4(2 + 3)^3$

**Homework:** “Order of Operations” Worksheet
3.4 – Using Exponents to Solve Problems

Examples:

(a) What is the surface area of a cube with an edge length of 5 cm?

(b) What is the volume of a cube with side length of 5 cm?

(c) Find the area of the square attached to the hypotenuse in the diagram, where \( a = 5 \text{ cm} \) and \( b = 12 \text{ cm} \).
(d) A circle is inscribed in a square with length of 20 cm. What is the area of the shaded region?

(e) The formula for the volume of a cylinder is \( V = \pi r^2 h \). Find the volume, \( V \), of a cylinder with radius of 6 cm and a height of 5.4 cm. Express your answer to the nearest tenth of a cubic centimeter.

(f) A pebble falls over a cliff. The formula that approximates the distance an object falls through air in relation to time is \( d = 4.9t^2 \), where \( d \) is distance, in metres, and \( t \) is time in seconds. What distance would the pebble fall during 4 s of free fall?

(g) A type of bacterium is known to triple every hour. There are 50 bacteria to start with. How many will there be after 5 hours?

**Homework:** Workbook: Pages 36 – 37 #4 – 11