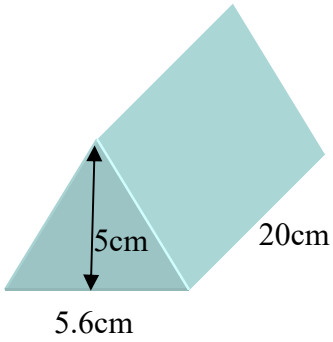
Date: KEY

7.4 Notes Solving Problems Involving Prisms and Cylinders

1) Toblerone Chocolate bars are sold in packages that are triangular prisms. Each bar measures 5.6cm along its base, is 5cm high and 20 cm long. Joe is going to stack the bars to make a triangular prism, but it must fit on a countertop that is only 50cm wide.



THINK! SOMETHING LIKE THIS:

FIND # OF BARS IN BOTTOM ROW:

$50cm \div 5.6cm = 8.92$

NINE WILL NOT FIT. MUST BE AN ODD-NUMBERED BASE, SO 7.

How many bars will fit in the shape?

$7 + 5 + 3 + 1 = 16$

What is the total volume of the display?

There are 16 bars in the display:

$V_D = 16V_B$

$= 16 A_{b_B} h_B$

$= 16\left(\frac{b_T h_T}{2}\right) h_B$

$= 16\left[\frac{(5.6)(5.0)}{2}\right](20)$

$V_D = 4480\ cm^3$

Alternatively:

(1) Find A_b of one bar:

$A_b = \frac{bh}{2}$

$A_b = \frac{(5.6)(5.0)}{2}$

$A_b = 14 cm^2$

(2) Find V of one bar:

$V = A_b h$

$V = (14)(20)$

$V = 280 cm^3$

(3) Find V of display (16 bars)

$V_D = 16V_B$

$V_D = 16(280)$

$V_D = 4480\ cm^3$

2) A section of pipe has an inner diameter of 12cm and an outer diameter of 15cm. What is the volume of a pipe that measures 5m long?

TWO METHODS:

① THINK OF THE PIPE AS TWO CYLINDERS. A SOLID OUTER CYLINDER AND AN INNER CYLINDER "CUT-OUT" OF THE OUTER.

$V = V_{out} - V_{in}$

$= \pi r_{out}^2 h - \pi r_{in}^2 h$

$= (3.14)(7.5)^2(500) - (3.14)(6)^2(500)$

$V = 31\,792.5\ cm^3$

② THINK OF IT AS ONE PRISM, $V = A_b h$, WITH A BASE LIKE THIS:

(a) $A_b = A_{out} - A_{in}$

$= \pi r_o^2 - \pi r_i^2$

$= (3.14)(7.5)^2 - (3.14)(6)^2$

$A_b = 63.585\ cm^2$

(b) $V = A_b h$

$V = (63.585)(500)$

$V = 31\,792.5\ cm^3$

3) Fred's truck has a bed that measures 2.5m x 1.2m x 2m. He has 105 m^3 of garbage he needs to haul to the dump. If each trip costs him $7, how much will it cost him to haul away all of his garbage?

ℓ h w

THINK BACKWARDS:

③ Cost = $7 × number of trips.

② Number of trips = $105\ m^3$ ÷ volume of bed (need to round properly)

① Volume of bed = ℓwh ⇐ WE CAN FIND THIS.

① $V = \ell wh$

$= (2.5)(2)(1.2)$

$V = 6\ m^3$

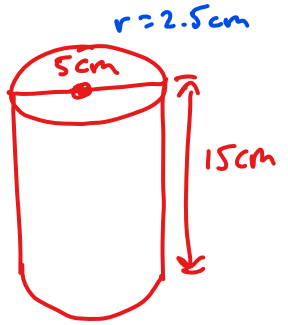
② Trips = $105 \div 6$

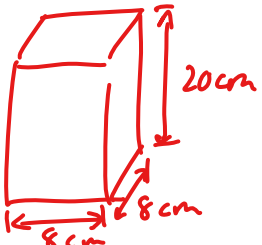
$= 17.5$

BUT YOU CAN'T MAKE HALF-TRIPS. HE'LL NEED 18 TRIPS.

③ $7 × 18 = $ $126

4) Simon has been hired to transport a canister of radioactive material. The canister is cylinder with a diameter of 5cm and a height of 15cm. He plans on putting it inside a rectangular box that measure 8cm on each side and 20cm high, and then filling the rest of the box with insulating foam to protect himself from the radioactivity. What volume of foam will he need?





$V_{foam} = V_r - V_c$

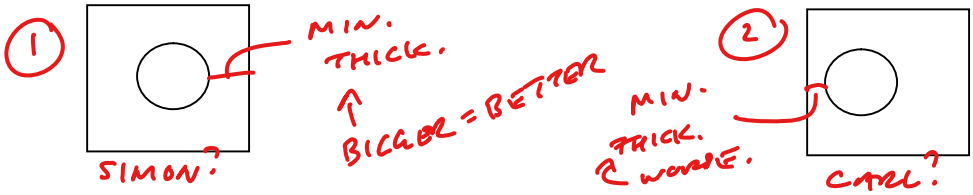
$= \ell w h - \pi r^2 h$

$= (8)(8)(20) - (3.14)(2.5)^2(15)$

$= 985.625$

$V_f = 985.6\ cm^3$

5) Simon's friend, Carl, has the brilliant idea of repositioning the canister within the box. He says to Simon that it will allow him to have more foam protection. Is he correct?



ANSWER: POSITION (1) IS BETTER.

EXPLANATION: SINCE INSULATION ACTS AS A BARRIER, AND RADIATION "RADIATES" OUTWARD IN ALL DIRECTIONS, THE BEST PROTECTION COMES FROM THE POSITION WITH THE BIGGEST MINIMUM "THICKNESS" OF INSULATION.